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### M427J Quiz 1 Solutions

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**Problem 1.** [2 pts] Find the general solution of

$$\frac{d\alpha}{dt} = \beta(t)\alpha(t).$$

$$\frac{\alpha'}{\alpha} = \beta \implies \log|\alpha| = \int \beta dt + C_1 \implies |\alpha| = e^{\int \beta dt + C_1} \implies \boxed{\alpha = Ce^{\int \beta dt}}$$

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**Problem 2.** [4 pts] Solve the initial value problem

$$tx'(t) + 2x(t) = \frac{1}{t \cos t}, \quad x(2\pi) = 1.$$

$$x' + \frac{2}{t}x = \frac{1}{t^2} \sec t, \quad \mu = e^{\int \frac{2}{t} dt} = t^2,$$

$$t^2 x' + 2tx = \sec t$$

$$\implies (xt^2)' = \sec t$$

$$\implies xt^2 = \log|\sec t + \tan t| + C$$

$$x(2\pi) = 1 \implies 1 \cdot (2\pi)^2 = \log|1 + 0| + C \implies C = 4\pi^2$$

$$\implies \boxed{x = \frac{\log|\sec t + \tan t| + 4\pi^2}{t^2}}$$

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**Problem 3.** [4 pts] Find the general solution of

$$\frac{y'}{t} - \cos^2(y)e^t = 0.$$

$$\frac{y'}{t} = \cos^2(y)e^t \implies \sec^2(y)y' = te^t$$

$$\implies \tan y = \int te^t dt = te^t - \int e^t dt = te^t - e^t + C$$

$$\implies \boxed{\tan y = (t-1)e^t + C}$$