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M427J Quiz 2 Solutions

Problem 1. [4 pts] Compute the Picard iterates y_1, y_2, y_3 for the initial value problem¹

$$y' = y, \quad y(0) = 1.$$

$$y_1 = 1 + \int_0^t 1 \, ds = \boxed{1 + t}$$

$$y_2 = 1 + \int_0^t (1 + s) \, ds = \boxed{1 + t + \frac{1}{2}t^2}$$

$$y_3 = 1 + \int_0^t (1 + s + \frac{1}{2}s^2) \, ds = \boxed{1 + t + \frac{1}{2}t^2 + \frac{1}{6}t^3}$$

Problem 2. [6 pts] Find the general solution to

$$y^2 - \frac{y}{t^2} + \left(y^2 - \frac{1}{t}\right) y' = 0.$$

$$M = y^2 - yt^{-2}, N = y^2 - t^{-1}$$

$$M_y = 2y - t^{-2}, N_t = t^{-2}$$

$$\frac{M_y - N_t}{N} = \frac{2y - 2t^{-2}}{y^2 - t^{-1}}, \text{ doesn't simplify to a function of just } t$$

$$\frac{N_t - M_y}{M} = -\frac{2y - 2t^{-2}}{y^2 - yt^{-2}} = -\frac{2}{y} \cdot \frac{y - t^{-2}}{y - t^{-2}} = -\frac{2}{y} \implies \mu(y) = e^{\int -2/y \, dy} = y^{-2}$$

Multiply by μ and DE becomes

$$1 - y^{-1}t^{-2} + (1 - y^{-2}t^{-1})y' = 0$$

Redefine

$$M = 1 - y^{-1}t^{-2}, N = 1 - y^{-2}t^{-1}$$

$$M_y = y^{-2}t^{-2} = N_t, \text{ (DE now exact)}^2$$

$$\implies \Phi = \int M \, dt = t + t^{-1}y^{-1} + f(y)$$

$$\implies \Phi_y = -t^{-1}y^{-2} + f'(y) = N = 1 - y^{-2}t^{-1}$$

$$\implies f'(y) = 1 \implies f(y) = y + C$$

$$\implies \boxed{t + t^{-1}y^{-1} + y = C}$$

¹Note the similarity to the Taylor series for the true solution $e^t = \sum_n \frac{1}{n!}t^n$.

²An unnecessary step to catch a possible mistake.