NAME: ______________________

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M427J Quiz 3

Problem 1. [2 pts] Give a judicious guess for the form of a solution to

$$y'' - 6y' + 9y = te^{3t}.$$

$$r^2 - 6r + 9 = (r-3)^2 \implies \boxed{y = (At^3 + Bt^2)e^{3t}}$$

Problem 2. [2 pts] Write the corresponding complex differential equation to

$$y'' + 4y' = \sin 4t.$$

Additionally, give a judicious guess for the form of a solution of the complex differential equation.

$$\boxed{y'' + 4y' = e^{4it}}$$

$$r^2 + 4r = r(r+4) \implies \boxed{y = Ae^{4it}}$$

Problem 3. [6 pts] Using variation of parameters, find the general solution to

$$y'' - y = t^{100}e^{-t}.$$

Your answer may contain at most one integral that has not been evaluated.

Since $r^2 - 1 = (r-1)(r+1)$, we have that $y_1 = e^t, y_2 = e^{-t}$ solve the homogeneous problem. We solve the system

$$y_1 u_1' + y_2 u_2' = 0$$
$$y_1' u_1' + y_2' u_2' = t^{100}e^{-t}.$$

which is

$$e^t u_1' + e^{-t} u_2' = 0$$
$$e^t u_1' - e^{-t} u_2' = t^{100}e^{-t}.$$

Adding and subtracting the equations gives

$$2e^t u_1' = t^{100}e^{-t} \implies u_1' = \frac{1}{2}t^{100}e^{-2t} \implies u_1 = \frac{1}{2}\int t^{100}e^{-2t}\,dt$$

$$-2e^{-t}u_2' = t^{100}e^{-t} \implies u_2' = -\frac{1}{2}t^{100} \implies u_2 = -\frac{1}{202}t^{101}$$

$$\boxed{y = \left(c_1 + \frac{1}{2}\int t^{100}e^{-2t}\,dt\right)e^t + \left(c_2 - \frac{1}{202}t^{101}\right)e^{-t}}$$