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### M427J Quiz 4 Solutions

**Problem 1. [3 pts]** Convert the following differential equation into a linear system of first order differential equations expressed in matrix form:

$$y'' - 6y' + 9y = 0, \quad y(4) = 3, \quad y'(4) = 5.$$

$$x_1 = y, x_1' = x_2$$

$$x_2 = y', x_2' = y'' = -9y + 6y' = -9x_1 + 6x_2$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -9 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \begin{pmatrix} x_1(4) \\ x_2(4) \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

or

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad x' = \begin{pmatrix} 0 & 1 \\ -9 & 6 \end{pmatrix} x, \quad x(4) = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

**Problem 2. [4 pts]** Compute the reduced row echelon form of

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 6 & 7 \end{pmatrix}.$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -4 & -8 & -12 & -16 \\ 0 & -2 & -4 & -6 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & -2 & -4 & -6 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**Problem 3. [3 pts]** Suppose that  $A$  is a matrix such that

$$\text{rref}(A) = \begin{pmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 1 & -8 & -9 \end{pmatrix}.$$

Give the complete solution set to the equation  $Ax = 0$ .

$$\left\{ \begin{pmatrix} -2\alpha - 4\beta - 5\gamma \\ \alpha \\ 8\beta + 9\gamma \\ \beta \\ \gamma \end{pmatrix} : \alpha, \beta, \gamma \in \mathbb{R} \right\}$$

or

$$\left\{ \alpha \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -4 \\ 0 \\ 8 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -5 \\ 0 \\ 9 \\ 0 \\ 1 \end{pmatrix} : \alpha, \beta, \gamma \in \mathbb{R} \right\}$$