NAME: ____________________

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M427J Quiz 7

Problem 1. **[4 pts]** In this question A is a real matrix. The parts should be considered separately. Use the given information to determine whether solutions to $\dot{x} = Ax$ are asymptotically stable, stable (but not asymptotically stable), or unstable. No work is required for this problem.

(a) A is 4×4 and has eigenvalues $-\sqrt{2}, -5, -\pi + 2i, -\pi - 2i$.
Choose one: asymptotically stable / stable / unstable

(b) A has characteristic polynomial $-\lambda^3$ and there are 3 linearly independent solutions to $Ax = 0$.
Choose one: asymptotically stable / stable / unstable

(c) A has characteristic polynomial $-(\lambda - 4)(\lambda^2 + 1)$.
Choose one: asymptotically stable / stable / unstable

(d) A is 3×3 and has only one eigenvalue -3. The equation $Av = -3v$ has only one linearly independent solution.
Choose one: asymptotically stable / stable / unstable

Problem 2. **[6 pts]** For $\alpha \neq 0$ the heat equation in two spacial dimensions is

$$u_t = \alpha^2(u_{xx} + u_{yy})$$

Assuming that $u(x, y, t) = X(x)Y(y)T(t)$, find ordinary differential equations satisfied by X, Y, and T. You may assume X, Y, and T are never zero. Hint: $F(t) = G(x, y)$ implies both sides are constant.

Problem (Bonus) **[2 pts]** The point of this question is to show that differentiable functions are not made up of too many high frequencies. Suppose f and f' are continuous on $[-1, 1]$. In this case the Fourier coefficients for f are given by

$$a_n = \int_{-1}^{1} f(x)\cos(n\pi x)\,dx, \quad b_n = \int_{-1}^{1} f(x)\sin(n\pi x)\,dx.$$

On the back of your quiz, show that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$. Hint: integrate by parts (differentiate f), then use that sine and cosine are bounded.[1]

[1] In fact, a much stronger statement is true: if the first $k \geq 0$ derivatives of f are continuous on $[-l, l]$, then the Fourier coefficients for f on $[-l, l]$ satisfy $\lim_{n\to\infty} n^k a_n = \lim_{n\to\infty} n^k b_n = 0$. That is, the smoother f is, the faster it's Fourier coefficients decay. This is much harder to prove.